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Effects of Large Mass Fermions on M_X and $\sin^2\theta_W$

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ABSTRACT

Some effects of heavy quarks and leptons are analyzed in the standard SU(5) model. We evaluate the two loop corrections to the SU(3), SU(2), and U(1) β-functions incorporating the effects of large Yukawa couplings of the fermions to the Higgs bosons. The corrections to M_X and $\sin^2\theta_W$ are found to be small for fermion masses less than $\sim$240 GeV.

I. INTRODUCTION

The renormalization group (RG) plays a central role in the standard grand unified theory of all quark and lepton interactions. Above the scale of electroweak symmetry breaking it is assumed the world may be described in terms of a symmetric, semisimple SU(3)×SU(2)×U(1) gauge theory with three distinct coupling constants, g_3, g_2 and g_1. The RG evolution of these coupling constants with increasing energy suggests that at some very large scale, $M_X \sim 10^{15}$ GeV, they merge into one coupling strength,[1] $g=g_1=g_2=g_3$ (where the "SU(5)" normalization of g_1 is employed) of some simple group, the most attractive candidate being the SU(5) model of Georgi and Glashow.[2] Hence, the RG supplied with a boundary condition of known experimental values of low energy coupling constants defines the energy scale of the grand unified symmetry breaking. Furthermore, the exact GUT symmetry relations between coupling constants at M_X fixes the low energy parameter $\sin^2\theta_W$ through the RG equations. Clearly any effect of new interactions that substantially modifies the RG equations threatens to displace the quantities M_X and $\sin^2\theta_W$ away from their canonical values and may violate known bounds on such quantities as τ_{proton}, etc.

Recently several authors have discussed the hypothetical existence of heavy fermions within the context of SU(5).[3,4,5] Such heavy objects have an effectively strong coupling to the mass-giving Higgs bosons and the RG dynamics of the Higgs-Yukawa coupling constants suggests that the masses may be determined by infrared fixed points of the RG equations. We emphasize that the Higgs-Yukawa coupling constants, g_H^f, are of order unity but that the perturbation theory is valid since, here, $(g_h^f)^2/16\pi^2$ is still very small. Pendleton and Ross[3] first noted that such a fixed point involving competition between g_H^{top} and g_{QCD} predicts a value of approximately 135 GeV for the mass of the top quark provided that g_H^{top} is near this fixed point at M_X. These authors also found an upper bound of order 220 GeV (in the absence of electroweak corrections) for m_{top} which is consistent with an earlier result of Cabibbo, Maiani, Parisi, and Petronzio[6] of $\sim$250 GeV including e.w. effects. Subsequently it was noted that there is an additional "intermediate fixed point" with a large domain of attraction such that an arbitrary, sufficiently heavy effective mass for a quark at M_X ($m \gtrsim 175$ GeV) will end up as a physical mass of $\sim$240 GeV.[4] However, this mechanism may be ruled out for the top-quark as it violates a bound due to Buras[7] of $m_t \lesssim 33$ GeV from $K_L \to \mu\bar{\mu}$ and the $K_S K_L$ mass difference and, furthermore, it undoes the successful prediction of m_b/m_τ in SU(5).[8] Veltman argues from a different point of view for a t-quark mass of order 69 GeV.[5]

The possibility of a fourth generation of quarks and leptons has been suggested by many authors and might indeed be welcome from an astrophysical point of view.[9] The intermediate fixed point predicts masses of $m_{+2/3} \sim 220$,

$m_{-1/3} \sim 215$ and $m_{-1} \sim 60$ GeV[4] to one loop accuracy and again raises the spectre of heavy fermions with large Higgs-Yukawa coupling constants. Here, the mass of a fermion with coupling constant g_H^f is given by $g_H^f\, v/\sqrt{2}$, where $v/\sqrt{2} \sim 175$ GeV, is the neutral Higgs boson vacuum expectation value and g_H^f is a RG effective coupling constant which is evaluated at (or near) the resulting fermion mass scale. If we considered models with additional Higgs bosons with vacuum expectation values v_i, then, for fixed light fermion masses, the Higgs-Yukawa couplings would become larger, since $v^2 = \sum_i v_i^2$, and hence $v_i \leq v$. In this case, even the lightest fermions might have large couplings to the Higgs bosons.

In any event, heavy fermions or several Higgs doublets may imply a new, relatively strong coupling to the Higgs bosons. For fermions in the standard model with one Higgs doublet with masses greater than ~ 175 GeV this becomes the largest coupling constant in the model over the range of the SU(5) desert. Hence, we inquire, how are the usual SU(5) predictions of $\sin^2\theta_W$ and M_X or τ_{proton} modified in the presence of heavy fermions?

II. RENORMALIZATION GROUP EQUATIONS

The RG evolution equations of g_1, g_2 and g_3 will now receive contributions from large Higgs-Yukawa coupling constants in two-loops. We may write these equations as follows:

$$16\pi^2 \frac{dg^i}{dt} = -\beta_0^i (g^i)^3 + \sum_j \frac{\beta_1^{ij}(g^j)^2(g^i)^3}{16\pi^2} + \sum_f \frac{c_f^i (g_H^f)^3 (g^i)^2}{16\pi^2} \quad (1)$$

where $t=\ln(\mu)$ and g_H^f is the Higgs-Yukawa coupling for a fermion species of type f. Here we have the usual results:

$$\beta_0^1 = -\frac{4}{3} N_g \quad \beta_0^2 = (22-4N_g)/3 \quad \beta_0^3 = (33-4N_g)/3$$

$$\beta_1^{ii} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -136/3 & 0 \\ 0 & 0 & 102 \end{bmatrix} + N_g \begin{bmatrix} 19/15 & 3/5 & 44/15 \\ 1/5 & 49/3 & 4 \\ 11/30 & 3/2 & 76/3 \end{bmatrix} \quad (2)$$

Our problem is the evaluation of the coefficients c_i^f in Eq.(1).

This evaluation is made simple by noting that one requires at least one fermion loop and one Higgs-boson line in each diagram contributing to the β-functions. For the nonabelian cases we evaluate the ghost-ghost-vector-boson vertex renormalization constant, the ghost wave function renormalization constant and the vector boson wave function renormalization constant to obtain β_i. We see immediately that only the vector-boson wave function renormalization constant contains such a configuration at the order $g_i^3(g_H^f)^2$. Hence, for SU(3) and SU(2) we need only consider the diagrams of Fig.(1). Similarly, for the U(1) case, we appeal to the Ward identity in calculating from the fermion-fermion-V.B. vertex, $Z_1=Z_2$, and again reduce our problem to the evaluation of only vacuum polarization diagrams as in Fig. (1). The calculation of these is straightforward.

Our results for the quantities c_f^i are presented in Table (I). Hence, for example, the extra effect of a +2/3 charged quark on the evolution of g_1 is a term of the form +(17/10) $g_H^2 g_1^3/(16\pi^2)$ in the RG equation for g_1 in Eq.(1). (Again, we are employing the SU(5) standard normalization g_1 where $g_1=\sqrt{5/3}\, g_1^{W.S.}$ where $g_1^{W.S.}$ is the Weinberg-Salam U(1) coupling constant).

Before embarking upon a detailed analysis of the combined two loop evolution of the g_i and the g_H^f it is useful to consider the <u>one loop</u> evolution of g_i with an assumed <u>constant</u> g_H^f. This is not a technically consistent approximation but it does indicate roughly the quantitive features of the effect upon M_X and $\sin^2\theta_W$. We recall that these quantities are determined by:

$$\alpha_3^{-1}(Q^2) - \alpha_2^{-1}(Q^2) = \frac{-(11 + n_H/2 + \delta_1)}{12\pi} \ln \frac{M_X^2}{Q^2} \qquad (3)$$

$$\sin^2\theta_W = \frac{3}{8}\left[1 - \frac{\alpha_{EM}(Q^2)}{4\pi}\left(\frac{5}{9}\right)\left(22 - \frac{n_H}{5} - \delta_2\right)\ln\frac{M_X^2}{Q^2}\right] \tag{4}$$

where δ_1 and δ_2 are the corrections from the inclusion of our present effects:

$$\delta_1 = \sum_f (c_2^f - c_3^f)\frac{(g_H^f)^2}{16\pi^2} \quad \delta_2 = \sum_f (c_2^f - c_3^f)\frac{(g_H^f)^2}{16\pi^2}\,. \tag{5}$$

Note that the value of M_X^2 in Eq. (4) must be determined from Eq. (3) including the effects of δ_1 for consistency. To this order α_3, and α_2 are determined from the experimental values of Λ_{QCD} and $\sin^2\theta_W$. α_{EM} has been extrapolated to $Q^2 \approx M_W^2$.

Suppose we have determined a set of values M_{Xo}^2 and $\sin^2\theta_{Wo}$ from the above Eq. (3) and Eq. (4) for a given set of input quantities in the approximation $\delta_1 = \delta_2 = 0$. Then, for small δ_1, δ_2, we find the corrections to these quantities are given by:

$$\delta M_X = -\frac{\delta_1 M_{Xo}}{(22+n_H)}\ln\left(\frac{M_{Xo}^2}{4m_f^2}\right) \tag{6}$$

$$\delta\sin^2\theta_W = -\frac{\alpha_{EM}}{4\pi}\left(\frac{5}{24}\right)\left\{\left(\frac{220-n_H}{110+5n_H}\right)\delta_1 - \delta_2\right\}\left(\ln\frac{m_{Xo}^2}{4m_f^2}\right) \tag{7}$$

where Q^2 is taken at the threshold for the heavy quark of mass $m_f = g_H^f \times (175\ \text{GeV})$. For example, let us consider a single +2/3 charged quark for which $\delta_1 = (-1/2)\ (g_H^f)^2/16\pi^2$ and $\delta_2 = -(4/3)\ \ ((g_H^f)^2/16\pi^2)$ and assume $n_H = 1$, $M_{Xo} = 10^{16}$ GeV and $\alpha_{EM} = 1/128$. Then we find:

$$\frac{\delta m_X}{m_X} \sim (8.99)\times 10^{-9}\ m^2_{+2/3}\{36.15 - \ln(m_{+2/3})\}$$

$$\frac{\delta \sin^2\theta_W}{\sin^2\theta_W} \sim -\ (.95)\times 10^{-10}\ m^2_{+2/3}\{36.15 - \ln m_{+2/3}\} \tag{8}$$

Thus, for a 10% increase in M_X we require such a quark to have a mass of $\sim$610 GeV and this would lead to a $-(.1)$% change in $\sin^2\theta_W$ (a $-1/3$ charged quark would produce the same effect in m_W^2 but would increase $\sin^2\theta_W$).

Thus our effects are expected to be small. However, it must be noted that g_H^f is not really a constant and increases with energy for quarks of mass greater than $\sim$175 GeV, hence these are under estimates of the effect. In fact, for quarks heavier than about 240 GeV, g_H^f actually diverges as we evolve upward before we reach M_X, as we discuss below.

Let us presently consider the evolution of the g_H^f coupling constants. We may use the one-loop evolution equations for the g_H^f to consistently study the two-loop effects upon the g_i. For a heavy fourth generation these equations are:[4]

$$16\pi^2 \frac{d}{dt} g_T = g_T\left(\frac{9}{2} g_T^2 + \frac{3}{2} g_B^2 + g_E^2 - 8g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{20} g_1^2\right)$$

$$16\pi^2 \frac{d}{dt} g_B = g_B\left(\frac{9}{2} g_B^2 + \frac{3}{2} g_T^2 + g_E^2 - 8g_3^2 - \frac{9}{4} g_2^2 - \frac{1}{4} g_1^2\right) \qquad (9)$$

$$16\pi^2 \frac{d}{dt} g_E = g_E\left(\frac{5}{2} g_E^2 + 3g_T^2 + 3g_B^2 - \frac{9}{4} g_2^2 - \frac{9}{4} g_1^2\right)$$

(where we've ignored the Cabibbo mixing to other generations; this turns out to be a small effect).

The fixed point behavior of these equations has been analyzed in Ref. (4) and it is found that, assuming $g_B(M_X) = g_E(M_X)$ (a consequence of SU(5) with a $\underset{\sim}{5}$ and $\underset{\sim}{24}$ of Higgs) there is a domain in the space spanned by g_T, g_B, g_E such that if a coupling constant lies within the domain boundary for $\mu \backsim 200$ GeV then it will remain finite for the entire evolution up to $M_X = 10^{15}$ GeV. However, if a coupling constant lies outside the domain boundary it will diverge before μ reaches M_X. Furthermore, there is a fixed point for which $m_T = g_T \times (175) \backsim 220$ GeV, $m_b = g_B \times (175) \backsim 215$ GeV and $m_E = g_E \times (175) \backsim 60$ GeV and this fixed point lies on the domain boundary. Hence, we will presently consider only those coupling constants that lie within the domain boundary. To analyze the values outside we must make additional, model dependent assumptions to effectively "clamp" the coupling constants of some large values before they diverge at M_X.

Let us now turn to the combined evolution of equations (9) and (1). Together, these form a system of six coupled, nonlinear, first order differential equations with the following boundary conditions

(a) at some point, M_X, g_1, g_2 and g_3 must meet to realize the SU(5) symmetry

(b) at some low energy, $\mu \sim M_W$, g_3 is fixed by the experimental value of Λ_{QCD} and $\alpha_{EM}^{-1} = 5/3\ \alpha_1^{-1} + \alpha_2^{-1}$, where α_{EM} is to be evaluated at μ ($\alpha_{EM}^{-1} = 127.5$[10,11]) and $\alpha_i = g_i^2/4\pi$.

(c) at μ, g_T and g_B are chosen so as to give the quarks the appropriate physical masses (these are assumed input parameters)

(d) at M_X, $g_B = g_E$.

Since the full coupled equations cannot be solved analytically and since the boundary conditions are not all uniformly applied at μ or M_X, an iterative-numerical scheme is needed to solve them. The following scheme we find converges rapidly:

(i) At μ we guess at a value of $\sin^2\theta_W$ and $m_E = g_E$ (175). α_{EM} and $\sin^2\theta_W$ determime α_1 and α_2, and g_3 is given by a choice of Λ_{QCD}.

(ii) Evolve upward until $\alpha_1 = \alpha_3$, This will determine a "first guess" for M_X which is a poor estimate if the initial guess for $\sin^2\theta_W$ is off by as much as .005. The evolved α_2 will generally not be equal to α_1 at this M_X.

(iii) At M_X, we fix α_2 at the value of α_1 and fix $g_E = g_B$.

(iv) Evolve back down to μ. The α_1 and α_2 now give a good value of $\sin^2\theta_W$. We find typically 3 iterations are sufficient.

III. RESULTS AND CONCLUSIONS

First we've analyzed the case of only three generations with a single heavy +2/3 charge (top) quark. Here we follow the discussion of Ref. (10) and assume $\Lambda_{\overline{MS}} = 400$ MeV, appropriately extrapolated through a b-quark to a heavy t-quark threshold in the threshold "theta-function" approximation. The single heavy t-quark has almost no effect upon $\sin^2\theta_W$ and M_X: a t-quark at 220 GeV changes $\sin^2\theta_W$ by $-.05\%$ and M_X by 1.1% while at 235 GeV, which is very near its fixed point value (and almost outside of the domain boundary), the changes are still only $-(.13)\%$ for $\delta\sin^2\theta_W/\sin^2\theta_W$ and $+2.9\%$ for $\delta M_X/M_X$ (these results appear in Table II).

The addition of a fourth generation, now assuming m_{top} is small and m_T, m_B and m_E to be heavy ~ 200 to 240 GeV, has the immediate effect of increasing M_X by a factor of 1.24 due to the additional fourth generation threshold which "stiffens" the evolution of α_3 relative to α_2 in two loops. We remark that this result is significantly smaller than those quoted in Ref. (10,11), who obtain ~ 1.8 as the increase in M_X, but is consistent with a result recently obtained by Marciano.[12] When the effects of the heavy quark Higgs-Yukawa couplings are now included the additional corrections to M_X and $\sin^2\theta_W$ are again negligible for g_T, g_B, and g_E inside of the domain boundary. These results appear in Table (III). The largest effects are of order $\delta\sin^2\theta_W/\sin^2\theta_W = -.1\%$ and

$\delta M_X/M_X = 3.6\%$. In fact, because of the specific values of the c_f^i in Eq. (1), if the T quark is away from its fixed point value, $\sin^2\theta_W$ remains unchanged (± 0.00001) even though B and E are near their fixed point values.

Hence, even for values of g_H^f near the fixed points at μ, where the g_H^f are diverging near M_X, they diverge so quickly at the high end of the desert that there is insufficient "evolution time" for them to have appreciable effects upon $\sin^2\theta_W$ and M_X. Essentially the effective averages of the g_H^f remain small and the analysis of Eq. (6) and Eq. (7) is approximately valid.

Suppose that there exist fermions which obtain mass from the standard Higgs bosons but which are heavier than $\backsim$240 GeV, or outside of the domain boundaries of Ref. (4). Then the g_H^f is diverging in perturbation theory at a scale $M'<M_X$ and the preceding discussion must be modified. Nonetheless, such objects may be composite or strongly coupled on a scale of M' and have constant strong couplings for scales $M'\leq\mu\leq M_X$. We have carried out the numerial experiment of "clamping" g_H^f at a large value of $g_H^f = 4\pi$ (hence $\alpha_H^f/4\pi = 1$) and keeping g_H^f constant at this value for all subsequant μ. In this case one does, of course, get large effects in $\sin^2\theta_W$ and M_X. For a single heavy t-quark one finds that $m_{top} \geq 380$ GeV gives a decrease in $\sin^2\theta_W$ greater than 5% and an increase in M_X greater than a factor of 3. Of course, the "clamping" scheme is neither a valid perturbative analysis nor likely a potentially realistic scenario, but it

does illustrate one of the pitfalls of having quark and lepton masses outside of the domain boundary or beyond the fixed points in grand unified models.

In conclusion, quark and lepton masses up to a scale of $\sim$240 GeV seem to be fully consistent with the standard grand unification scenario and and do not appreciably alter the standard results for M_X and $\sin^2\theta_W$ obtained in the approximation of neglecting large Higgs-Yukawa coupling constants. Heavier objects obtaining their masses from the usual isodoublet Higgs are problematic for grand unification in a) destroying the successful $\sin^2\theta_W$ and wiping out proton decay b) and creating strong coupling limits in the desert from the usual RG evolution.

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TABLE CAPTIONS

Table 1: C_f^i, the contribution to the β function proportional to $(g_H^f)^3 g_i^2$.

Table 2: The effects of a heavy top quark on $\sin^2\theta_W$ and M_X (3 generations).

Table 3: The effects of a fourth generation of heavy quarks (through g_H becoming large) on $\sin^2\theta_W$ and M_X.

TABLE I

$\frac{i}{f}$	SU(3)	SU(2)	U(1)
up	2	3/2	17/10
down	2	3/2	1/2
electron	0	1/2	1/2
neutrino	0	1/2	3/10

TABLE II.

M_{top} (GeV)	$\Delta \sin^2\theta/\sin^2\theta$ (%)	$\Delta M_X/M_X$ (%)
0	0	0
50	-.001	.02
100	-.004	.09
150	-.011	.24
200	-.026	.60
220	-.042	.97
235*	-.109	2.54

TABLE III.

M_T	M_B	$\Delta\sin^2\theta_W/\sin^2\theta_W$ (%)	$\Delta M_X/M_X$ (%)
0	0	0	0
30	30	.000	.02
100	30	-.003	.10
200	30	-.023	.56
242*	30	-.099	2.40
30	100	.000	.17
100	100	-.003	.25
200	100	-.024	.78
240*	100	-.120	1.83
30	200	.002	.84
100	200	-.003	1.00
200	200	-.041	2.59
209*	203*	-.084	4.91
30	228*	.003	1.46

FIGURE CAPTIONS

Figure 1: Diagrams contributing to the β-function for g_H. The solid line is a fermion, the dashed line the Higgs scalar, and the wavy line a gauge boson. Diagrams a and c contribute in SU(3), Diagrams b,c, and d in SU(2), and all four in U(1).

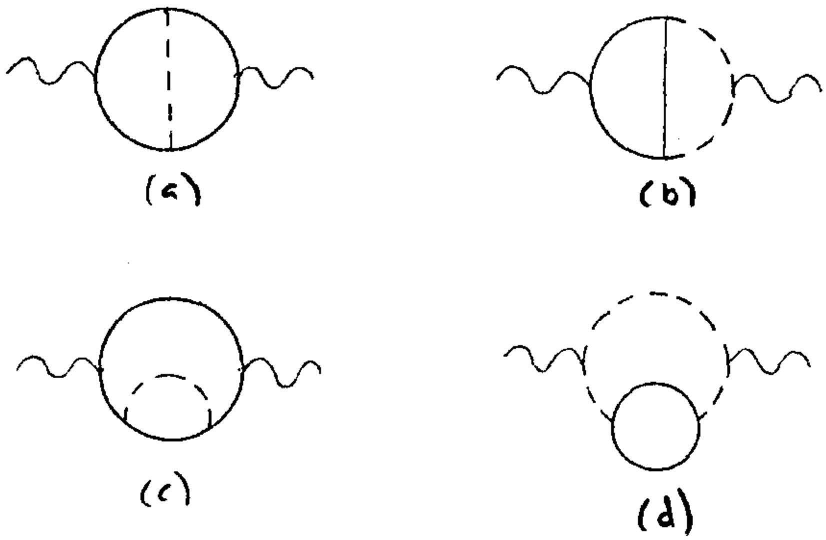

Fig. 1